Part I

CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called a circular motion with respect to that fixed (or moving) point.

ANGULAR VELOCTIY (ω)

Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} \; ; \quad \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

Instantaneous Angular Velocity

The rate at which the position vector of a particle with respect to the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relative Angular Velocity

$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

here $V_{AB_\perp}$ = Relative velocity perpendicular to position vector AB

Relation between speed and angular Velocity : $v = r\omega$ is a scalar quantity ($\vec{\omega} \neq \frac{\vec{v}}{r}$)

ANGULAR ACCELERATION (α)

Average Angular Acceleration

Let ω_1 and ω_2 be the instantaneous angular speed at time t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_1 - \omega_2}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

It is the limit of average angular acceleration as Δt approaches zero, that is

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

Part II

RADIAL AND TANGENTIAL ACCELERATION

$a_t = \frac{dv}{dt}$ = rate of change of speed and $a_r = \omega^2 r = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$

Angular and Tangential Acceleration Relation

$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$ or $a_t = r\alpha$

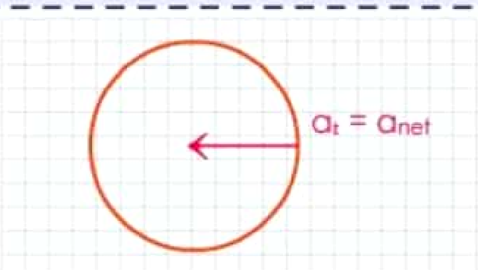
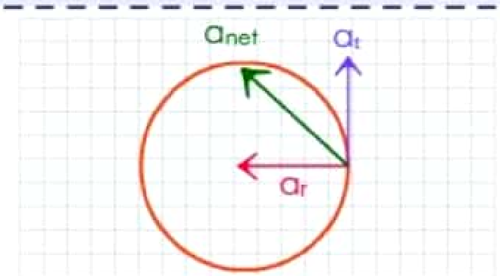
RELATIONS AMONG ANGULAR VARIABLES

Equations of Rotational Motion

$$\omega = \omega_0 + \alpha t$$

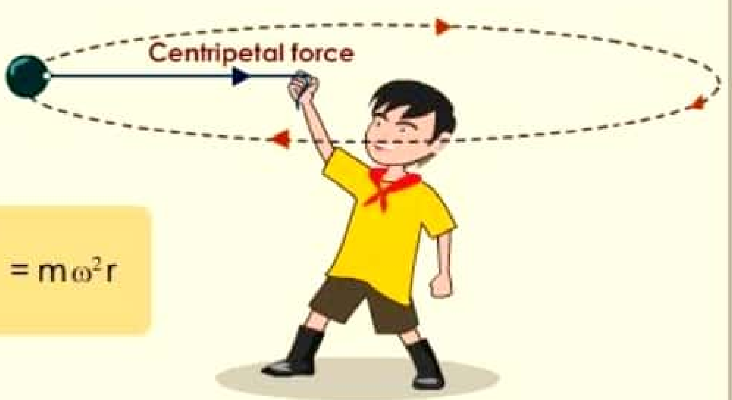
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

Uniform Circular Motion	Non-Uniform Circular Motion
Speed of the particle is constant i.e., ω = constant	Speed of the particle is not constant i.e., $\omega \neq$ constant
$a_t = \frac{d\lvert\vec{v}\rvert}{dt} = 0$; $a_r = \frac{v^2}{r} \neq 0$ $\therefore$ $a_{net} = a_r$	$a_t = \frac{d\lvert\vec{v}\rvert}{dt} \neq 0$; $a_r \neq 0$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
	

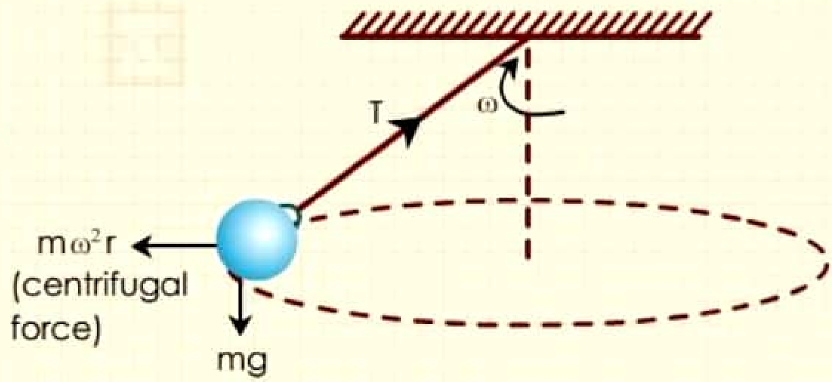
CENTRIPETAL FORCE

Centripetal force is the necessary resultant force towards the centre.



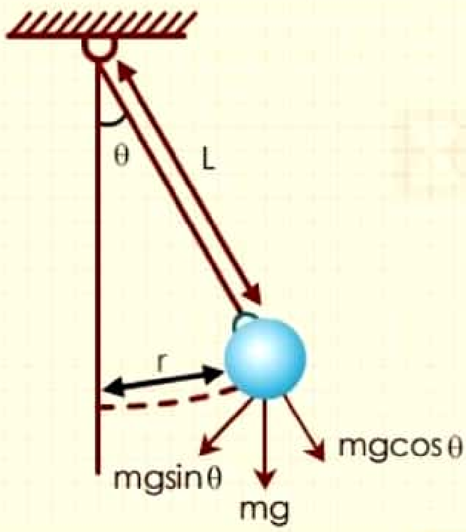
$$F = \frac{mv^2}{r} = m\omega^2 r$$

CENTRIFUGAL FORCE



Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion (in that frame)

$$F_c = m\omega^2 r$$



SIMPLE PENDULUM

Balancing Horizontal Forces:

$$T \sin\theta = m\omega^2 r$$

Balancing Vertical Forces:

$$T - mg\cos\theta = mv^2/L \Longrightarrow T = m(g\cos\theta + v^2/L)$$

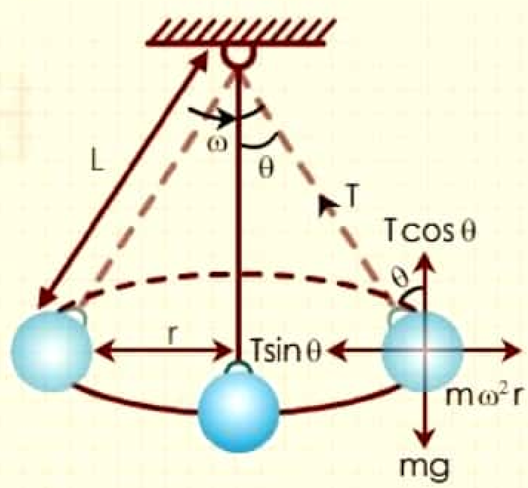
$$|\vec{F}_{net}| = \sqrt{(mg\sin\theta)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\theta + \frac{v^4}{L^2}}$$

CONICAL PENDULUM

FBD of ball shows:

$T\sin\theta = m\omega^2 r$ = centripetal force

$T\cos\theta = mg$



FBD of ball w.r.t ground

speed $v = \frac{r\sqrt{g}}{(L^2 - r^2)^{1/4}}$ and Tension $T = \frac{mgL}{(L^2 - r^2)^{1/2}}$